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AN INVENTORY MODEL FOR TIME DEPENDENT DETERIORATE RATE AND VARIABLE HOLDING COST

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ABSTRACT

This paper intends to show about the positive reflectance of an inventory model by using quadratic demand, shortage allow, deterioration rate time dependent with variable Holding cost. Finally we will see the effect of those parameters in total inventory cost. A numerical example is also taken to discuss the sensitivity of the models.

KEYWORDS: Inventory Model, Deterioration Rate, Demand, Shortages, Time Dependent & Holding Cost

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1. INTRODUCTION

Inventory is described as stock of goods. It also described as the process to raise the business. It comprises all the features like storage of goods/products, controls the number of products in stock. Inventory is ideal for run the business successfully and effectively. If inventory not applicable, it may harm the business. It comprises various divergent conditions into models. This condition may comprises the deterministic demand, change in demand with time, deterioration etc.

In inventory models, deterioration may be taken as degeneracies loss of resources. The fact of controlling and maintaining inventory of deteriorating items become a challenging for decision making.

Demand plays very important role in inventory. In the classical inventory model the demand rate was assumed to be constant but it is not always possible. For example seasonal fruits, vegetables, other items etc.

Inventory models can be divided into two kinds of models. First is deterministic and another one is stochastic model. In deterministic model, demand for a time period is known. In stochastic model, the demand is a random variable having known probability distribution.

By using the deterioration of resources, Inventory for deteriorating goods was first studied by Whitin in 1952. Emmons's (1968), Azoury and Miller (1984), Azoury (1985) expanded the approach. In the present paper, we consider the model of Covert and Philip (1973) and extend it to include a time-quadratic demand rate, shortages in inventory and time dependent holding cost. Demand changing with time was expanded by Dave and Patel S.K. Ghosh and K.S. Chaudhary had considered quadratic demand in their inventory model. An inventory model of deteriorating items was developed by Hariya (1995) for time varying demand with shortages. Chang and Dye (1999) developed an inventory model with time varying demand. Uttam Kumar Khedelkar and Diwahar Shukla

and Raghovendra Pratap Singh Chandel, proposed a logarithmic inventory model with a shortage of deteriorating items. In (2005) we studied in inventory system for the product with price time dependent demand. Later Ghosh and Chaudhuri (2004, 2006) Khara and Chaudhuri (2003) established their models with quadratic time varying demand. Vinod Kumar et.al studied an inventory model for deteriorating items with time dependent demand and time- varying holding cost under partial backlogging. An inventory model for deteriorating items with price dependent demand and time varying holding cost was developed by Ajanta Roy in 2008.

In this paper, we studied about the inventory models for deteriorating products having quadratic demand with respect to time and variable holding cost with respect to time. Here, we take the deterioration rate is proportional to time in inventory model. This sensitivity analysis is done by numerical examples.

2. ASSUMPTIONS AND NOTATIONS

This mathematical model is described by using the following assumptions and notations

Assumptions are,

1. The demand rate $f\ (t)$ at times it is assumed as $f(t) = a + bt + ct^2$; a, b, c are constants.
2. Replenishment occurs.
3. Shortages are allowed.
4. $\theta(t) = \theta t$ is deterioration rate.
5. Here holding cost is time dependent i.e. $C_H = \alpha + \beta t, where\ \alpha > 0, \beta > 0$

Notations are,

1	C_S	=	Shortage cost per unit per unit time.
2	C_O	=	Ordering cost per order.
3	C_D	=	Deterioration cost.
4	W	=	The maximum inventory level for each ordering cycle.
5	S	=	The maximum amount of inventory
6	Q	=	The order quantity ($Q = W + s$).
7	I(t)	=	Inventory level at time t.
8	t_1	=	Time at which shortages start.
9	T	=	Total length of each ordering cycle.
10	TIC	=	Total inventory cost over the period (0, T).
11	α	=	Holding cost parameter
12	β	=	Holding cost parameter

3. MATHEMATICAL FORMULATION

In this paper, we are assuming the replenishment of a deteriorating product with shortages and variable holding cost. Here we note out the optimal order quantity which is Q and total optimal inventory cost. The below graph (figure 1) represent the behaviour of inventory at any time.

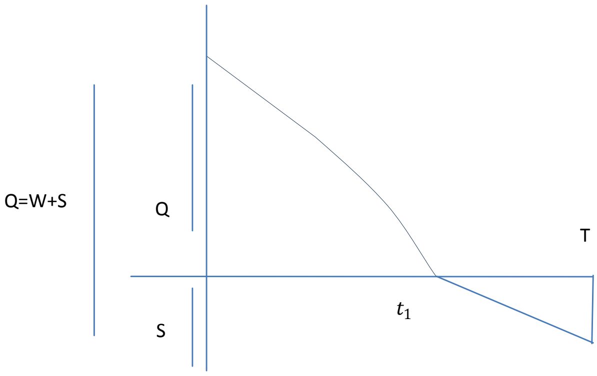

Figure 1

The inventory level maximum at t=0 and replenishment is made. After this inventory level is decreased within the time period [0, t], and ultimately falls to zero at $t = t_1$. Further at t_1 shortages occur during the time interval $[t_1, T]$.

Now till the shortages are allowed at interval $[0, t_1]$, the differential equation is given by:

$$\frac{dI_1(t)}{dx} + \theta t I_1(t) = -(a + bt + ct^2); 0 \leq t \leq t_1 \quad (1)$$

And during the interval $[t_1, T]$, the shortage occurs,so the differential equation is given by:-

$$\frac{dI_2(t)}{dx} = -(a + bt + ct^2); t_1 \leq t \leq T \quad (2)$$

With the boundary conditions: $t = 0,\ I(0) = W$,

$t = t_1\,;\ I(t_1) = 0$

$t = T\,;\ I(T) = S$

Now, by solving above equations (1), we get,

$$I_1(t) = a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{c}{3}(t_1^3 - t^3) + \frac{a\theta}{6}(t_1^3 - t^3) + \frac{b\theta}{8}(t_1^4 - t^4) + \frac{c\theta}{10}(t_1^5 - t^5) - \frac{a\theta}{2}(t^2 t_1 - t^3) - \frac{b\theta}{4}(t^2 t_1^2 - t^4) - \frac{c\theta}{6}(t^2 t_1^3 - t^5) - \frac{a\theta^2}{12}(t^2 t_1^3 - t^5) - \frac{b\theta^2}{16}(t^2 t_1^4 - t^6) - \frac{c\theta^2}{20}(t^2 t_1^5 - t^7) \quad (3)$$

By solving equation (2) we get:

$$I_2(t) = a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{c}{3}(t_1^3 - t^3) \quad (4)$$

Now, at t=0 the maximum inventory level for each cycle is given by

$I(0) = W, t = 0$

$$W = I_1(0) = at_1 + \frac{b}{2}t_1^2 + \frac{c}{3}t_1^3 + \frac{a\theta}{6}t_1^3 + \frac{b\theta}{8}t_1^4 + \frac{c\theta}{10}t_1^5$$

And at t=T the maximum amount of quadratic demand per cycle is given by

$t = T,\ I_2(t) = -S$

$$S = -a(t_1 - T) - \frac{b}{2}(t_1^2 - T^2) - \frac{c}{3}(t_1^3 - T^3)$$

Now, the order quantity per cycle is,

$$Q = W + s = at_1 + \frac{b}{2}t_1^2 + \frac{c}{3}t_1^3 + \frac{a\theta}{6}t_1^3 + \frac{b\theta}{8}t_1^4 + \frac{c\theta}{10}t_1^5 - a(t_1 - T) - \frac{b}{2}\left(t_1^2 - T^2\right) - \frac{c}{3}\left(t_1^3 - T^3\right) \tag{5}$$

Holding cost per unit per unit time is given by,

$$HC = \int_0^{t_1} (\alpha + \beta t) I_1(t)$$

$$HC: \frac{1}{2}\alpha a t_1^2 + \left(\frac{1}{3}\alpha b + \frac{1}{6}\beta a\right)t_1^3 + \left[\alpha\left(\frac{1}{4}c + \frac{1}{12}a\theta\right) + \frac{1}{8}\beta b\right]t_1^4 + \left[\frac{1}{15}\alpha b\theta + \beta\left(\frac{1}{10}c + \frac{1}{40}a\theta\right)\right]t_1^5 + \left[\alpha\left(\frac{1}{18}c\theta - \frac{1}{72}a\theta^2\right) + \frac{1}{48}\beta b\theta\right]t_1^6 + \left[\beta\left(\frac{1}{56}c\theta - \frac{1}{112}a\theta^2\right) - \frac{1}{84}\alpha b\theta^2\right)t_1^7 - \left(\frac{1}{96}\alpha c\theta^2 + \frac{1}{128}\beta b\theta^2\right)t_1^8 - \frac{1}{144}\beta c\theta^2 t_1^9 \tag{6}$$

Shortages cost per unit per unit time is given by

$$SC = (-)\, C_S \int_{t_1}^{T} I_2(t)$$

$$SC: -C_S\left[a\left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) + b\left(\frac{t_1^2 T}{2} - \frac{T^3}{6} - \frac{t_1^3}{3}\right) + c\left(\frac{t_1^3 T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4}\right)\right] \tag{7}$$

Ordering cost per order is given by:

$$OC = C_O \tag{8}$$

Now, deteriorating cost is given by,

$$DC: C_D\left[W - \int_0^{t_1} f(t)dt\right]$$

$$DC = C_D\left[\frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8} + \frac{c\theta\, t_1^5}{10}\right] \tag{9}$$

Therefore, the total cost per unit time per unit cycle is given by,

$$TIC = \frac{1}{T}(HC + SC + OC + DC)$$

$$TIC: \frac{1}{T}\left\{\frac{1}{2}\alpha a t_1^2 + \left(\frac{1}{3}\alpha b + \frac{1}{6}\beta a\right)t_1^3 + \left[\alpha\left(\frac{1}{4}c + \frac{1}{12}a\theta\right) + \frac{1}{8}\beta b\right]t_1^4 + \left[\frac{1}{15}\alpha b\theta + \beta\left(\frac{1}{10}c + \frac{1}{40}a\theta\right)\right]t_1^5 + \left[\alpha\left(\frac{1}{18}c\theta - \frac{1}{72}a\theta^2\right) + \frac{1}{48}\beta b\theta\right]t_1^6 + \left[\beta\left(\frac{1}{56}c\theta - \frac{1}{112}a\theta^2\right) - \frac{1}{84}\alpha b\theta^2\right)t_1^7 - \left(\frac{1}{96}\alpha c\theta^2 + \frac{1}{128}\beta b\theta^2\right)t_1^8 - \frac{1}{144}\beta c\theta^2 t_1^9 - C_S\left[a\left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) + b\left(\frac{t_1^2 T}{2} - \frac{T^3}{6} - \frac{t_1^3}{3}\right) + c\left(\frac{t_1^3 T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4}\right)\right] + C_O + C_D\left[\frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8} + \frac{c\theta\, t_1^5}{10}\right]\right\} \tag{10}$$

Our motive of this paper is to find out the optimal values of t_1 and T in order to minimize the total inventory cost. In this paper, there are two variables so we get the optimal values by equating their partial derivative to zero. This is the essential condition to minimize the total inventory cost.

$$\frac{d(TIC)}{dT} = \frac{-1}{T^2}\Bigg[\frac{1}{2}\alpha a t_1^2 + \left(\frac{1}{3}\alpha b + \frac{1}{6}\beta a\right)t_1^3 + \left[\alpha\left(\frac{1}{4}c + \frac{1}{12}a\theta\right) + \frac{1}{8}\beta b\right]t_1^4 + \left[\frac{1}{15}\alpha b\theta + \beta\left(\frac{1}{10}c + \frac{1}{40}a\theta\right)\right]t_1^5 + \left[\alpha\left(\frac{1}{18}c\theta - \frac{1}{72}a\theta^2\right) + \frac{1}{48}\beta b\theta\right]t_1^6 + \left[\beta\left(\frac{1}{56}c\theta - \frac{1}{112}a\theta^2\right) - \frac{1}{84}\alpha b\theta^2\right)t_1^7 - \left(\frac{1}{96}\alpha c\theta^2 + \frac{1}{128}\beta b\theta^2\right)t_1^8 - \frac{1}{144}\beta c\theta^2 t_1^9 - C_S\left[a\left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) + b\left(\frac{t_1^2 T}{2} - \frac{T^3}{6} - t_1\right) + c\left(\frac{t_1^3 T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4}\right)\right] + C_O + C_D\left[\frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8} + \frac{c\theta\, t_1^5}{10}\right] -$$

$$\left.\frac{C_S\left[a(t_1-T)+b\left(\frac{t_1^2}{2}-\frac{T^2}{2}\right)+c\left(\frac{t_1^3}{3}-\frac{T^3}{3}\right)\right]}{T}\right] \tag{11}$$

$$\frac{d(TIC)}{dt_1}=\frac{1}{T}\Big\{\alpha a t_1+3\left(\frac{1}{3}\alpha b+\frac{1}{6}\beta a\right)t_1^2+4\left[\alpha\left(\frac{1}{4}c+\frac{1}{12}a\theta\right)+\frac{1}{8}\beta b\right]t_1^3+5\left[\frac{1}{15}\alpha b\theta+\beta\left(\frac{1}{10}c+\frac{1}{40}a\theta\right)\right]t_1^4+ 6\left[\alpha\left(\frac{1}{18}c\theta-\frac{1}{72}a\theta^2\right)+\frac{1}{48}\beta b\theta\right]t_1^5+7\left[\beta\left(\frac{1}{56}c\theta-\frac{1}{112}a\theta^2\right)-\frac{1}{84}\alpha b\theta^2\right)t_1^6-8\left(\frac{1}{96}\alpha c\theta^2+\frac{1}{128}\beta b\theta^2\right)t_1^7- \frac{1}{16}\beta c\theta^2 t_1^8-C_S\left[a(T-1)+b\left(t_1T-t_1^2\right)+c\left(t_1^2T-t_1^3\right)\right]+C_D\left[\frac{a\theta t_1^2}{2}+\frac{b\theta t_1^3}{2}+\frac{c\theta\, t_1^4}{2}\right]\Big\} \tag{12}$$

We get the optimal values of t_1 and T by solving equation (11) & (12) by using MAPLE 15.

4. NUMERICAL EXAMPLE

Now we take a numerical example to check how the solution is optimal. We will solve the example with the help of Maple 15.

To explain the model numerically, the following parameters of the inventory system are:

$a=18, b=14, c=12, C_S=6, C_0=80, C_D=12, \theta=0.001, \alpha=0.05, \beta=20$

Under the above given parameters by using Maple 15 we get the optimal shortage value

$t_1=0.4153283522$ per unit time and optimal length of ordering cycle is $T=1.113048332$ unit time.

Total inventory cost is $TIC=151.5165463$.

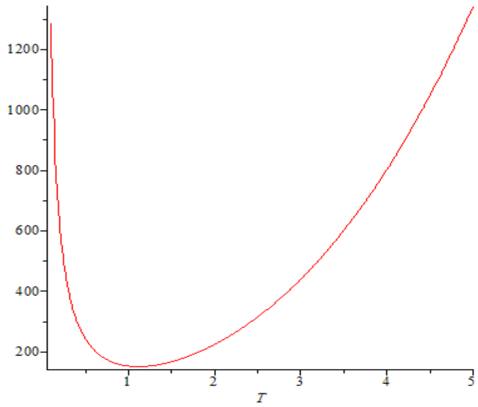

Figure 2

5. SENSITIVE ANALYSIS

Table 1

Parameter	% Change	Change in			
		T	t_1	Q	TIC
a	+20%	0.9545182953	0.13912536800	30.47400806	163.9357624
	+10%	1.0944646740	0.39135899860	35.29965172	157.4315731
	-10%	0.9615548808	0.05351289470	25.60547162	148.3074235
	-20%	1.1453148810	0.4574304186	31.68453756	139.4979320
b	+20%	0.9400200144	0.08895323602	27.66546680	155.9701463
	+10%	1.1010842790	0.41304338260	34.49434880	152.7860765
	-10%	0.9618753103	0.07185131890	26.70226663	152.2496011
	-20%	1.1387396940	0.4203285997	33.66581703	148.8791134

Table 1: Contd.,					
c	+20%	0.9488389161	0.09226431529	27.48149899	154.5442793
	+10%	0.9516538368	0.08532957604	27.26147268	154.0310201
	-10%	0.9576913344	0.07141985096	26.82078002	152.9892273
	-20%	1.1377634420	0.42861490530	34.25468253	150.0814910
C_S	+20%	1.0696836600	0.4200190349	32.97596120	167.35474065
	+10%	0.9415335914	0.1158791036	27.04806896	161.32884820
	-10%	1.139760291	0.4114843429	36.51884717	145.16031920
	-20%	1.000	0.000	29.66666667	140.0000
C_O	+20%	0.9901969640	0.01328941313	29.21774071	170.9510697
	+10%	0.9713940766	0.04295369287	28.36786946	162.7386767
	-10%	1.067513701	0.3783082545	32.86966222	145.2700448
	-20%	1.005004948	0.3143386247	29.89753546	137.4126604
C_D	+20%	1.1130284390	0.4152872055	35.14128166	152.7347842
	+10%	1.1130383850	0.4153077780	35.14178822	152.7344980
	-10%	0.9546013003	0.0783099549	27.62117362	154.3808358
	-20%	1.113068228	04153695054	35.14330818	152.7336393
α	+20%	1.1126810380	0.41452659390	34.20521187	151.5342649
	+10%	0.9546375109	0.07845811616	27.04277598	153.5130407
	-10%	1.1132318250	0.41572897130	34.23189769	151.5076603
	-20%	1.1134152150	0.41612941810	34.24078574	151.4987562
β	+20%	0.9558034375	0.08097398286	27.64724365	154.3906122
	+10%	0.9551804576	0.07963015339	27.64673378	154.3855869
	-10%	0.9540612434	0.07721593368	27.59735064	154.3763365
	-20%	0.9535545697	0.07612302273	27.57501144	154.3720546
θ	+20%	0.9546023169	0.07838305902	27.62121837	154.3808443
	+10%	1.1130376110	0.4153062393	35.14177724	152.7345104
	-10%	1.1130590550	0.4153504674	35.14281255	152.7339133
	-20%	0.9546009722	0.07838015834	27.62115844	154.3808330

- With increase or decrease in a and b, TIC increases or decreases respectively and Q vary with it.
- If c increases, TIC increases and length of cycle i.e. T decreases.
- If CS (shortage cost) and CO(ordering cost) increases, TIC increases.
- If CD (deterioration cost) decreases, TIC and Q varies with it.
- If holding parameter α decreases, there is flexibility in TIC and Q.
- If β increases, TIC and Q increases.
- TIC and Q vry with increase and decrease in θ.

6. CONCLUSIONS

Here we converse the model in case of quadratic demand (function of time) and assumed that deterioration rate and holding cost is changing with time. By analytical solution of above model we get minimum total inventory cost. Finally, this model has been demonstrated by the numerical and graphical analysis. The acquire results specify the stability of the model. The above model is very thoughtful in case of time dependent demand and holding cost. This model further can be expanded for other form of demand rate.

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